Monopoles and the Chiral Phase Transition in $SU(2)$ Lattice Gauge Theory

Roy Wensley [a] *

[a]Department of Mathematical Sciences, Saint Mary's College, Moraga, California, 94575, USA

In the quenched approximation we use the abelian and monopole fields from abelian projection in SU(2) lattice gauge theory to numerically compute the value of the chiral condensate. The condensate calculated using abelian projection is observed to vanish at the same critical temperature as the full SU(2) theory predicts.

1. Introduction

Calculations of the $SU(2)$ string tension using abelian projected fields and their monopole contributions have given good evidence that confinement in non-abelian theories can be explained by abelian degrees of freedom and that the monopoles in these configurations are the confinement mechanism[1]-[6]. Color confinement is the not the only non-perturbative effect to be understood in gauge theories. Chiral symmetry breaking is well known to have no perturbative explanation and it seems clear that presence of a non-zero chiral condensate is related to topology in non-abelian theories[7,8]. Is it possible for the effective abelian theory, which seems to give an explanation of confinement in terms of magnetic monopoles, to explain the chiral phase transition? In this paper we present results which demonstrate that the chiral phase transition in quenched $SU(2)$ lattice gauge theory is reproduced in abelian projected gauge fields after fixing to the maximal abelian gauge.

2. Calculation of the Chiral Condensate

The order parameter used to study the chiral symmetry transition is the chiral condensate defined by

$$\langle \bar{\psi}\psi(m)\rangle = \frac{1}{V}\mathrm{Tr}(\not{D}(U)+m)^{-1},$$

where V is the lattice volume, m is the input mass of the quenched fermion, and U is the lattice gauge field. Spontaneous chiral symmetry breaking is observed in the limit $V \to \infty$ when $\langle \bar{\psi}\psi(0)\rangle \neq 0$, i. e. when the chiral condensate remains finite at zero bare quark mass. To study the chiral phase transition numerically, it is most straightforward to use staggered fermions, since chiral symmetry is maintained explicitly on the lattice. Using staggered fermions leads to the Dirac eigenvalue equation

$$\begin{aligned} i\not{D}[U]\psi &= \sum_{\mu} \frac{i\eta_\mu(x)}{2}(U_\mu(x)\psi(x+\hat{\mu}) \\ &\quad -U^\dagger_\mu(x-\hat{\mu})\psi(x-\hat{\mu})) \\ &= \lambda_n \psi(x) \end{aligned} \quad (1)$$

The calculation of the chiral condensate is done in the following way: First, we use the Lanczos method to calculate a small set of the lowest eigenvalues of Eqn. (1). These lowest eigenvalues are then used to compute the spectral density function $\rho(\lambda)$. Finally, the Banks-Casher formula[9]

$$\langle \bar{\psi}\psi(0)\rangle = \pi\rho(0)$$

is used to extract the chiral condensate at zero bare quark mass.

In Eq. (1) The link variable $U_\mu(x)$ in Eq. (1) can come from any gauge group. In the work done here, Eq. (1) is used to compute eigenvalues from both $SU(2)$ configurations and from abelian configurations generated using abelian projection.

*This work was supported in part by the National Science Foundation, Grant No. PHY-9403869, the Higher Education Funding Council for Wales, and the Saint Mary's College Faculty Development Fund.

3. The Monte Carlo Calculations

$SU(2)$ gauge field configurations were generated using a heatbath Monte Carlo simulation on a $16 \times N_\tau$ lattice at $\beta = 2.5115$ for values of $N_\tau =$ 4, 6, 8, 12, and 16. After allowing for 5000 equilibrium updates, 100 configurations separated by 40 updates were used for measurements. The abelian projection was done by fixing to the maximum abelian gauge. The method used for gauge fixing and extracting abelian fields is described in Ref. [5].

3.1. Boundary Conditions

In computing fermionic observables at finite temperature it is important to consider the boundary conditions. To enforce Fermi statistics for the fermionic fields on the lattice, anti-periodic boundary conditions in the τ-direction should be used. For the fundamental gauge group representation it is known (numerically) that the chiral restoration and deconfinement transitions occur at the same critical temperature[10]. Chiral restoration is signaled by a vanishing chiral condensate while deconfinement is signaled by a non-vanishing Polyakov line value. At the value of β used in this study, the phase transitions are known to occur at $N_\tau = 8$[11].

The deconfinement phase in pure $SU(2)$ is also accompanied by the spontaneous breaking of a discrete global Z_2 symmetry. This breaking of the Z_2 symmetry is reflected in the sign of the average value of the Polyakov line $\langle P \rangle$. The global sign of the Polyakov line will affect the boundary condition when solving Eq. (1). In order to maintain anti-periodic boundary conditions (APBC) it is necessary to implement periodic boundary conditions (PBC) when $\langle P \rangle < 0$ and APBC when $\langle P \rangle > 0$ (see Ref. [10] for a discussion). The subtlety with boundary conditions is expected to be an issue only for the quenched approximation and only in the chirally symmetric phase, i. e. only for $N_\tau < 8$.

4. Results

The spectral density function was calculated for the full $SU(2)$, abelian projected, and monopole gauge fields. The 25 lowest eigenvalues from the 100 configurations at each N_τ were used. The average value of the Polykov line $\langle P \rangle$ was monitored for each configuration. In all cases it was found that the value $\mathrm{sgn}(\langle P \rangle)$ was the same for the full $SU(2)$, abelian, and monopole fields of each configuration. For the case $N_\tau = 4$ it was found that $\langle P \rangle < 0$, thus PBC were used in computing the eigenvalues. While, for $N_\tau = 6$ it was found that $\langle P \rangle > 0$, and so APBC were used.

In Fig. (1), the results for $N_\tau = 4$ are shown. It seems clear that all three functions are approaching the same intercept, however the detailed behavior does not agree. Here we are considering the physics of the chiral condensate and so we are only interested in the intercept. In Fig. (2), $\rho(\lambda)$ is shown for the abelian fields at each value of N_τ used that corresponds to a finite temperature. A constant+linear+quadratic fit was used to characterize the functions, and the intercept, $\rho(0)$, was extracted as a measure of the condensate.

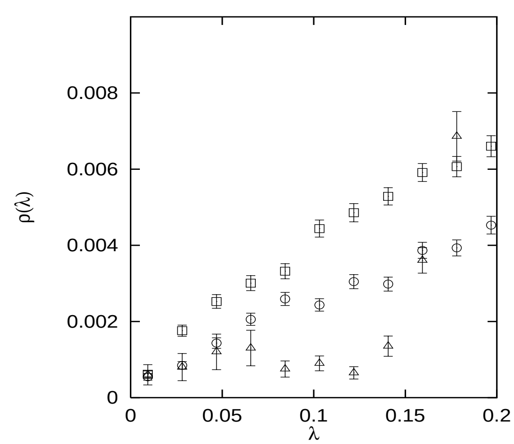

Figure 1. A comparison of the spectral density function at $N_\tau = 4$ for full $SU(2)$ (triangles), abelian (squares), and monopole (circles) fields.

As a check that the boundary conditions used did indeed make a difference for configurations above the critical temperature, calculations at

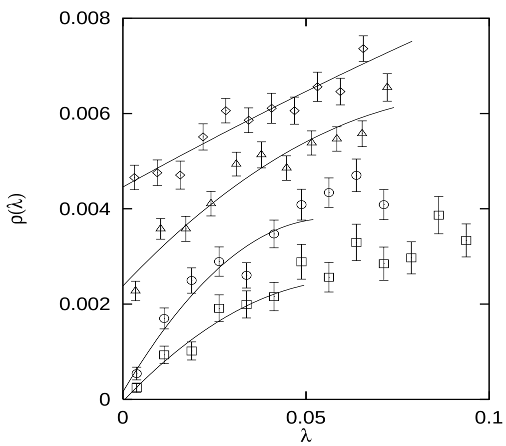

Figure 2. The spectral density function $\rho(\lambda)$ for small eigenvalues with $N_\tau = 4$ (squares), 6 (circles), 8 (triangles), and 12 (diamonds). The results for zero temperature ($N_\tau = 16$) are not shown for the sake of clarity, as they lie on top of the $N_\tau = 12$ points.

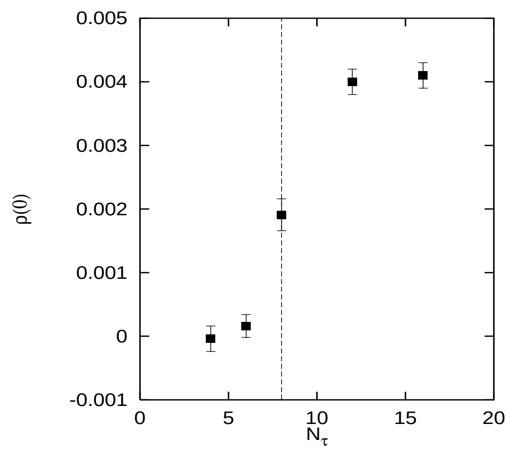

Figure 3. The value of $\rho(0)$ as a function of inverse temperature N_τ. The dashed line is the accepted location of the phase transition.

$N_\tau = 4$ using APBC were also done. In these calculations, the behavior of $\rho(\lambda)$ was found to be more like the symmetrically broken case, and the intercept was consistent with a non-zero value. The value of $\rho(0)$ for the APBC was found to be .0048(2). This can be compared to the value found for the PBC case (shown in Fig. (1)) which was found to be -.00004(20), which is consistent with zero.

In Fig. (3), the value of $\rho(0)$ (using the correct boundary conditions) as a function of N_τ is presented. It is clear from the figure that chiral symmetry is restored at $N_\tau = 8$. This corresponds to the accepted critical temperature of previous studies using full $SU(2)$ gauge fields[11].

5. Conclusions

The results presented indicate that the chiral phase transition can be observed using the abelian projected fields in $SU(2)$. This gives evidence that non-perturbative effects in non-abelian theories may be explained by an effective abelian theory, and thus ultimately tied to magnetic monopoles.

REFERENCES

1. T. Banks, J. Kogut, and M. Myerson, Nucl. Phys. **B129** (1977) 493.
2. J. D. Stack and R. J. Wensley, *Nucl. Phys.* **B371** (1992) 597
3. G. 't Hooft, Nucl. Phys. **B190** (1982) 455.
4. A. S. Kronfeld, M. Laursen, G. Schierholz and U.-J. Wiese, Phys. Lett. B **198** (1987) 516.
5. J. Stack, S. Neiman, and R. Wensley, Phys. Rev. **D50** (1994) 3399.
6. H. Shiba, T. Suzuki, Phys. Lett. **B333** (1994) 461.
7. G. 't Hooft, *Phys. Rev.* **D14** (1976) 3432.
8. S. J. Hands and M. Teper, *Nucl.Phys.* **B347** (1990) 819.
9. T. Banks and A. Casher *Nucl. Phys.* **B169** (1980) 103.
10. J. B. Kogut, M. Stone, H. W. Wyld, S. H. Shenker, J. Shigemitsu, and D. K. Sinclair, *Nucl. Phys.* **B225[FS9]** (1983) 326.
11. G.S. Bali, J. Fingberg, U.M. Heller, F.

Karsch, K. Schilling *Phys.Rev.Lett.***71** 3059 (1993).